SIMPLE SPREADSHEET THERMAL MODELS FOR CRYOGENIC APPLICATIONS

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ABSTRACT

Self consistent circuit analog thermal models that can be run in commercial spreadsheet programs on personal computers have been created to calculate the cooldown and steady state performance of cryogen cooled Dewars. The models include temperature dependent conduction and radiation effects. The outputs of the models provide temperature distribution and Dewar performance information. These models have been used to analyze the SIRTF Telescope Test Facility (STTF). The facility has been brought on line for its first user, the Infrared Telescope Technology Testbed (ITTT), for the Space Infrared Telescope Facility (SIRTF) at JPL. The model algorithm as well as a comparison between the models' predictions and actual performance of this facility will be presented.

INTRODUCTION

The current economic climate dictates that the designed performance of large scale cryogenic systems be traded off against cost to minimize the resources necessary to develop the system while still meeting the technical requirements. In keeping with NASA's desire to simplify and reduce the cost of flight operations, the Jet Propulsion Laboratory (JPL) undertook the task of a quick development, low cost facility for the optical interferometric testing of mirrors of diameter $\leq$ 1 m, f $\leq$ 6, at temperatures from 300 to 5 K. The project was constrained by an allocation of one year in which to complete the design, fabrication, and optical qualification test, with funding commensurate with the allotted schedule.

In order to meet these design challenges, self consistent circuit analog thermal models that could be run in a commercial spreadsheet program on a personal computer were developed to calculate the cooldown and steady state performance of cryogen cooled Dewar designs. These models were used to identify design aspects where the resources necessary to develop the system could be minimized while still meeting the technical requirements.

MODEL DESCRIPTIONS

Steady State Model

The equations for one dimensional conductive and radiative heat transfer across an area A,

$$Q = \lambda(T)\, A \frac{\partial T}{\partial x} \tag{1}$$

and

$$Q = \varepsilon\, \sigma_{SB}\, A\, (T_2^4 - T_1^4) \tag{2}$$

can be written in terms of effective thermal resistances R

$$Q = R^{-1} \Delta T \tag{3}$$

where

$$\Delta T = (T_2 - T_1) \tag{4}$$

The conductive resistance is

$$R_C = \left[\frac{A}{l}\, \bar{\lambda}\right]^{-1} \tag{5}$$

where l is the length of conductive segment and $\bar{\lambda}$ is the average conductivity of the segment

$$\bar{\lambda} = \int_{T_1}^{T_2} \lambda(T)\, dT / \Delta T \tag{6}$$

The radiative resistance is

$$R_R = \frac{\Delta T}{\varepsilon\, \sigma_{SB}\, A\, (T_2^4 - T_1^4)} \tag{7}$$

A network of conductive and radiative heat transfer paths that model an entire thermal system can then be solved by treating the system as an electrical resistor network

$$Q = R^{-1} \Delta T \Rightarrow I = R^{-1} V \tag{8}$$

Although the radiative and conductive thermal resistances are highly temperature dependent, a self consistent steady state solution to the circuit analog thermal model can be found by executing the following steps:

- An initial distribution of temperatures at each resistor network node point is assumed (consistent with the room temperature and cryogen boiling points). $\{T_1, T_2, \ldots, T_i, T_j, \ldots, T_N\}$
- The temperatures at each resistor network node point, and the known thermal properties of the link (conductive or radiative) are used to calculate the (thermal) resistance of each link in the system network.

$$R_{ij}^{C} = \left[\frac{A}{l\,(T_j - T_i)} \int_{T_i}^{T_j} \lambda(T)\, dT\right]^{-1} \tag{9}$$

$$R_{ij}^{R} = \frac{T_j - T_i}{\varepsilon_{ij}\, \sigma_{SB}\, A\, (T_j^4 - T_i^4)} \tag{10}$$

- Using Kirkoff's rules, the (heat) current through each resistive element is calculated (using room temperature and the cryogen boiling points as points of fixed "potential").

- The calculated heat currents and thermal resistances are used to calculate a new distribution of temperatures at the resistor network node points (using room temperature and the cryogen boiling points as fixed points).

$$T_j - T_i = R_{ij} \cdot Q_{ij}$$

- The steps are repeated with the system temperature distribution calculated in the previous step until the heat flow though each element calculated by the network solution and the heat flow though each element calculated from thermal properties of the individual element differ by less than 0.1%.

This series of steps can be implemented on a personal computer within a spreadsheet program which can invert matrices. To calculate the network solution to the heat flow in each resistive element, Kirkoff's laws are used to write a matrix **K** of equations multiplying the element currents. The matrix **K** can be written as three sub-matrices: K_1, K_2 and K_3. The matrix K_1 multiplied by the vector of element currents **Q** is a set of equations of all thermal paths between room temperature and cryogen fixed points and is equal to a sub matrix ΔT. The matrix K2 multiplied by the vector of element currents is set of equations for all closed loops between fixed points within the resistor network and is equal to a null vector **0**. The matrix K_3 multiplied by the vector of element currents is the set of equations for the difference between the sum of the currents going into each node and the sum of the currents out of each node. It is also equal to null vector **0**.

$$[K]\begin{bmatrix} Q_1 \\ \vdots \\ Q_{ij} \\ \vdots \\ Q_N \end{bmatrix} = \begin{bmatrix} [K_1] \\ [K_2] \\ [K_3] \end{bmatrix}_{ij} Q_{ij} = \begin{bmatrix} [\Delta T] \\ [0] \\ [0] \end{bmatrix} \tag{12}$$

The network solution to the heat flow though each element can then be calculated by inverting the matrix of equations obtained from Kirkoff's laws K.

$$\begin{bmatrix} Q_1 \\ \vdots \\ Q_{ij} \\ \vdots \\ Q_N \end{bmatrix} = [K]^{-1}\begin{bmatrix} [\Delta T] \\ [0] \\ [0] \end{bmatrix} \tag{13}$$

Time Dependent Cooldown Model

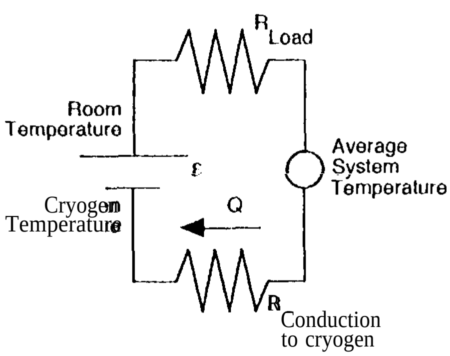

Figure 1 Equivalent resistor circuit model for the average system temperature and cryogen heat load

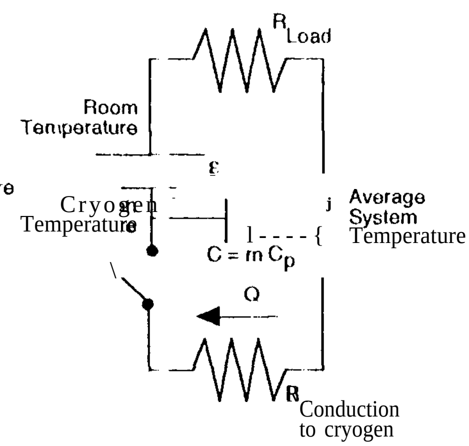

Figure 2 Equivalent RC circuit model for the average system temperature and cryogen heat load

From the information obtained with the steady state model it is possible to create a time dependent cooldown model for the average cooling rate of the entire system by executing the following steps:

- Create an equivalent resistor circuit model for the average system temperature T and cryogen heat load Q (Fig. 1).
- The time dependent cooldown behavior can then be modeled with an RC circuit obtained by adding a capacitor in parallel to, and a switch in series with, the thermal resistance to the cryogen bath (Fig. 2).

The average system temperature is then given by

$$T_{system}^{\ avg.} = T_{cryogen} + \tilde{\varepsilon}\,(1 + p\ \exp^{-t/\tilde{R}C}) \tag{14}$$

where

$$\tilde{\varepsilon} = \frac{\varepsilon R_C}{(R_L + R_C)} \tag{15}$$

$$p = \frac{R_L}{R_C} \tag{16}$$

$$\tilde{R} = \frac{R_L R_C}{(R_L + R_C)} \tag{17}$$

$$C = m\ C_p \tag{18}$$

where R_L and R_C are the thermal load and conduction to cryogen resistances, respectively, and m and C_p are the mass and specific heat of the cooled parts 01 the system, respectively.

'[he heat load to the cryogen is given by

$$Q = \frac{\tilde{\varepsilon}}{R_C}(1 + p\ \exp^{-t/\tilde{R}C}) \tag{19}$$

DEWAR STRUCTURE

Tbc STTF Dewar has a vertically oriented inner cylindrical test chamber with a diameter 01 1.4 m and an internal height 01"2.3 m. The experiment mounting surfaces, the upper and lower surfaces of the chamber, are the exterior surfaces 01' two liquid helium tanks. The upper liquid helium tank may be positioned at any height within the cylindrical wall of the test chamber to facilitate the testing of optics with various focal lengths. The cylindrical wall of the test chamber is conductively cooled by the two 300 liter capacity tanks. Surrounding the test chamber is bell shaped aluminum radiation shield, This bell is conductively cooled through a bolted contact joint with a 300 liter capacity liquid nitrogen tank directly below the helium cooled test chamber. The outer vacuum shell of the Dewar is constructed of an aluminum dome that makes an O - Ring seal with a stainless steel lower shell surrounding the lower cryogen tanks. Structural and thermal separation between the outer vacuum shell, the nitrogen tank, and the lower helium tank is provided by titanium struts and a stainless steel tension/compression system necessary for earthquake safe operation of the facility in southern California. The entire Dewar had to be designed to withstand 1.0 g in the vertical and 0.5 g in the horizontal axis without serious hazard to human safety. Optical access is provided through a window at the bottom 01 the Dewar, with cryogenically cooled baffles and shutters in the apertures through the lower cryogen tanks. The Dewar was fabricated through a partnering arrangement with Janis Research Company, Inc.

The nitrogen cooled bell shaped radiation shield and LN2 tank were covered with multilayer insulation (MLI) blankets that were constructed of layers of aluminized mylar with spacer material[1]. The following equation[2]

$$\frac{Q}{A} = \frac{\varepsilon \frac{\sigma_{SB}}{100}\left(T_2^{4.67} - T_1^{4.67}\right)}{n} + C \frac{N^{2.56}}{1+n} \frac{\left(T_2^2 - T_1^2\right)}{2} \qquad (20)$$

was used [o model the heat transfer through the MLI in W/m2 where C = 8.85 x 1()8, N = # Layers/cm, and n = # Layers. Ten layers was chosen to meet the design performance requirements while minimizing fabrication cost and schedule. All exterior helium cooled surl'aces (tank walls and shroud) were covered with a single layer of al uminized mylar to , -d i s c the surface emissivity[3].

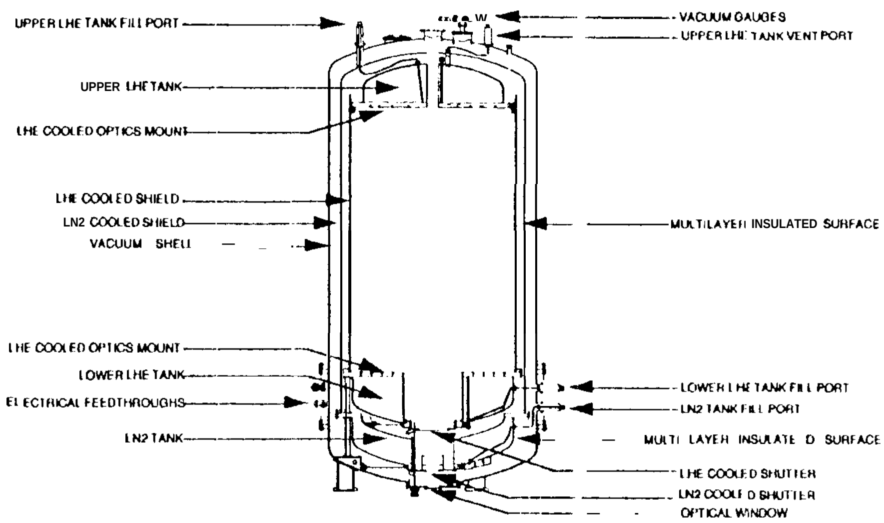

Figure 4 STTF Dewar structure

COMPARISON TO **SYSTEM** PERFORMANCE

"Initial estimate models" were run during the design phase of the project, and "data enhanced **models"** have been run subsequent to the operation of the facility, using data obtained during the operation to calculate effective model parameters. Results for both sets of models are presented below

Liquid nitrogen cool down

After achieving a guard vacuum of better than 10^{-4} Torr in the Dewar, the cool down of the STTF begins with the introduction of liquid nitrogen into all three cryogen tanks simultaneously. Cool down is achieved by first cooling the tank walls adequately to accumulate liquid inside and fill the tanks, with the conductively cooled surfaces then relaxing to equilibrium with a longer time constant. Cool down 10 steady state took about 72 hours and required approximatel y 3000 liters of liquid nitrogen when transferring at an average rate of 9 liters/hour into each tank.

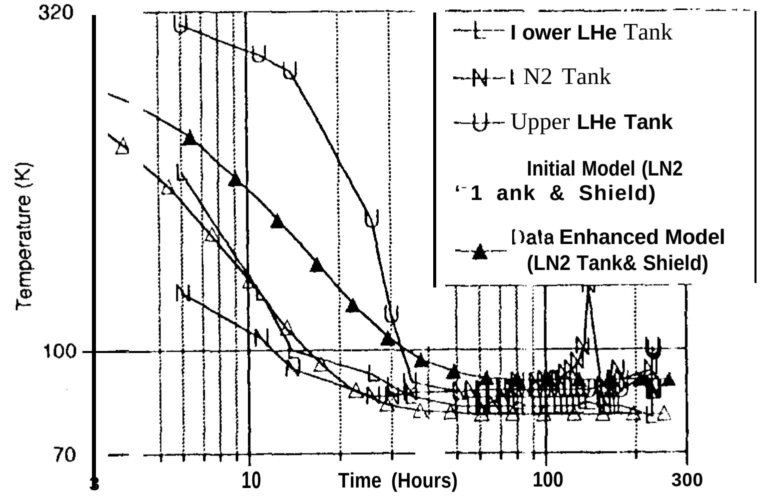

Figure 5 Temperatures within the STTF as a function time during the liquid nitrogen cool down

The model is consistent with the approach to steady state for the nitrogen tank and radiation shield. It is beyond the scope of this simple model to predict the LN_2 pre-cool 01 the test chamber (helium tanks & shroud), but mass and construction similarities would indicate any difference should be no more than 20 %.

Liquid nitrogen steady state performance

At steady state, with liquid nitrogen in all three tanks, the parasitic heat load (as measured by cryogen boil off rate) to the upper helium, lower helium and nitrogen tanks were 2.4 W, 2.8 W and 99 W, respectively. This is well within an order of magnitude of the original estimate, and is easily modeled by adjusting the loft (layers/cm) of the MLI.

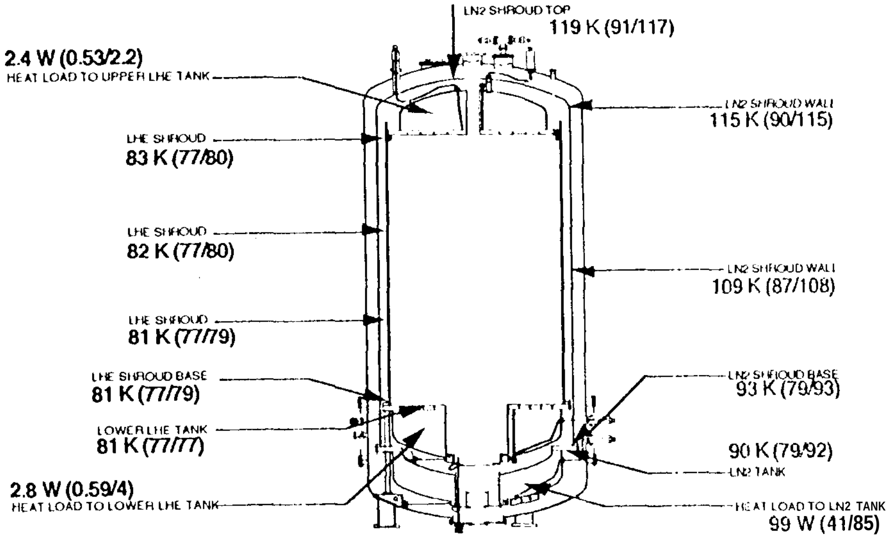

Figure 6. Steady state performance of the STTF with liquid nitrogen in all three tanks In parenthesis are the initial estimate / data enhanced model values.

Liquid helium cool down

To cool the test chamber below liquid nitrogen temperatures, any remaining liquid nitrogen is purged from both liquid helium tanks by over pressurizing the tanks with helium gas. Liquid helium is [hen transferred into both tanks simultaneously. Cool down is achieved by first cooling the tank walls adequately [o accumulate liquid inside and fill the tanks, with the conductively cooled surfaces then quickly relaxing to equilibrium. Coot down to steady stale took about 24 hours and required approximately 1200 liters of liquid helium when transferring at an average rate of 1 liter/minute into each tank. Although the model correctly predicts the total time required for the test chamber to reach steady state, it dots not correctly model the precipitous drop in temperature that occurs when liquid starts accumulating in the tanks.

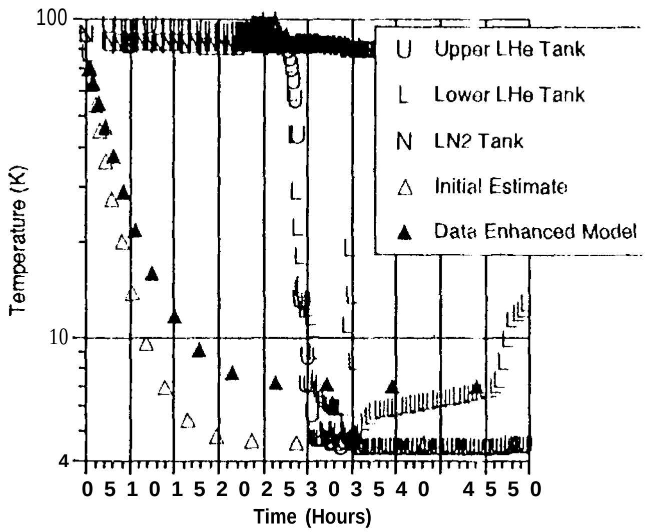

Figure 7 Temperatures within the STTF as a function time during the first liquid helium cool down 1 he transfer of liquid helium began at O hours.

Liquid helium steady state performance

At steady state, with liquid helium in both the upper and lower helium tanks, and liquid nitrogen in the nitrogen tank, the parasitic heat load (as measured by cryogen boil off rate) to the upper helium, lower helium and nitrogen tanks were 2.2 W, 9.7 W and 57 W, respectively. This is within an order of magnitude of' the original estimate, and is easily modeled by adjusting the loft (layers/crrl) of the MLI. The data enhanced thermal model has shown that the thermal profile and the difference in the boil off rates in the upper and lower helium tanks arc consistent with a factor of 4 difference in the thermal conductivity of the links between the helium shroud and the upper and lower helium tanks respectively. A minor modification has been made to the upper tank mounting, scheme that should provide over 36 hours of cryogen lifetime in both helium tanks. The thermal model has also verified that the thermal profile and heat loads are consistent with radiation dominated heat loads. This thermal model indicates that over 95% 01 the heat load to the nitrogen cooled surfaces and over 66% of the heat load to the helium cooled sulfates are radiative in nature.